

**33.52** A 25ft high water tank supplies a fountain with its outlet at ground level via gravity through a short length of 3in diameter steel pipe. The transition from the tank to the piping has a sharp entrance, and the outlet has a threaded 90-degree elbow such that the fountain discharges vertically upward. What is the maximum height achieved by the fountain stream?

- A. 10.9ft
- B. 11.4ft
- C. 13.9ft
- D. 24.2ft

Write the **Bernoulli Equation** where State 1 is the top of the tank and State 2 is fountain outlet at ground level.

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f$$

The tank and the outlet are both open to the atmosphere, therefore  $P_1 = P_2$ . The velocity in the supply tank is negligible, so  $v_1 \approx 0$ . The outlet is at ground level, so  $z_2 = 0$ . Simplify the equation.

$$z_1 = \frac{v_2^2}{2g} + h_f$$

The connection from the tank to the outlet is a “short length” of pipe so it is valid to neglect the major friction losses. However, there is a **Sharp Entrance** and an elbow so the minor losses dominate the  $h_f$  term. Use the **Threaded Pipe Fittings** table to obtain the K-Factor for the elbow.

$$h_f \approx h_{f,minor} = K \frac{v^2}{2g}$$

$$h_f = (0.5 + 0.8) \frac{v^2}{2g} = 1.3 \frac{v^2}{2g}$$

Substitute into the previous equation along with the height,  $z_1$ , and solve for the exit velocity,  $v_2$ .

$$z_1 = \frac{v_2^2}{2g} + 1.3 \frac{v_2^2}{2g}$$

$$z_1 = 2.3 \frac{v_2^2}{2g}$$

$$25ft = 2.3 \frac{v_2^2}{2g}$$

$$v_2 = \sqrt{\frac{(25ft)(2)\left(32.2\frac{ft}{s^2}\right)}{2.3}} = 26.46\frac{ft}{s}$$

Having solved for the exit velocity, determine the maximum height the stream will reach. It is valid to formally write out the Bernoulli equation again considering the max height as  $z_3$ . Everything cancels except the velocity at State 2 and the height at State 3.

$$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{v_3^2}{2g} + z_3 + h_f$$

$$\frac{v_2^2}{2g} = z_3$$

$$z_3 = \frac{\left(26.46\frac{ft}{s}\right)^2}{2\left(32.2\frac{ft}{s^2}\right)} = 10.9ft$$

**Answer A**